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Wang, L., Kim, K. J., Duong, T. Q., El Kashlan, M., & Poor, H. V. (2014). Security Enhancement of Cooperative Single Carrier Systems. In *IEEE Transactions on Information Forensics and Security* (Vol. 10, pp. 90-103). Institute of Electrical and Electronics Engineers Inc.. <https://doi.org/10.1109/TIFS.2014.2360437>

**Published in:**

IEEE Transactions on Information Forensics and Security

**Document Version:**

Peer reviewed version

**Queen's University Belfast - Research Portal:**

[Link to publication record in Queen's University Belfast Research Portal](#)

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# Security Enhancement of Cooperative Single Carrier Systems

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**Abstract**—In this paper, the impact of multiple active eavesdroppers on cooperative single carrier systems with multiple relays and multiple destinations is examined. To achieve the secrecy diversity gains in the form of opportunistic selection, a two-stage scheme is proposed for joint relay and destination selection, in which, after the selection of the relay with the minimum effective maximum signal-to-noise ratio (SNR) to a cluster of eavesdroppers, the destination that has the maximum SNR from the chosen relay is selected. In order to accurately assess the secrecy performance, the exact and asymptotic expressions are obtained in closed-form for several security metrics including the secrecy outage probability, the probability of non-zero secrecy rate, and the ergodic secrecy rate in frequency selective fading. Based on the asymptotic analysis, key design parameters such as secrecy diversity gain, secrecy array gain, secrecy multiplexing gain, and power cost are characterized, from which new insights are drawn. Moreover, it is concluded that secrecy performance limits occur when the average received power at the eavesdropper is proportional to the counterpart at the destination. Specifically, for the secrecy outage probability, we confirm that the secrecy diversity gain collapses to zero with outage floor. For the ergodic secrecy rate, we confirm that its slope collapses to zero with capacity ceiling.

**Index Terms**—Cooperative transmission, frequency selective fading, physical layer security, secrecy ergodic rate, secrecy outage probability, single carrier transmission.

## I. INTRODUCTION

Due to the broadcast nature of wireless channels and the emergence of smart cities and the Internet of Things (IoT) in the digital economy, the wireless infrastructure is continuously being exposed to security threats of eavesdropping that may potentially intercept or interrupt the communication between the legitimate terminals. As such, security and privacy are of utmost concern for future wireless technologies. Security was originally implemented as a high-layer problem to be

solved using cryptographic methods. However, in some network architectures, such cryptographic security is practically infeasible due to high complexity in data encryption and decryption and the distributed nature of the infrastructure. Alternatively, in wireless physical (PHY) layer security, the breakthrough idea is to exploit the characteristics of wireless channels such as fading or noise to transmit a message from a source to an intended destination while keeping the message confidential from passive eavesdroppers [1]. Different from cryptographic methods, where the eavesdroppers can intercept the information exchange and then try to decrypt the cryptographic protection by quantum computer, the main idea of PHY layer security is to exploit the uncorrelated nature of the wireless medium with the aim to maximize the uncertainty of the legitimate information at the eavesdropper.

It is not until recent years that the concept of PHY layer security has attracted considerable interest amongst wireless network designers and marked widespread adoption in the study of information security for radio communication systems. The key idea is to degrade the signal-to-noise ratio (SNR) of the eavesdropper relative to the legitimate receiver. This will guarantee perfect secrecy in wiretap channels. Driven by this and with the aid of multiple-input multiple-output (MIMO) technology, PHY layer security in MIMO wiretap channels that employ multiple colocated antennas at the transmitter, the legitimate receiver, and/or the eavesdropper has attracted considerable attention (e.g., [2]–[7], and the references therein). Maximal ratio combining (MRC) for security enhancement was proposed in [2] and the secrecy outage probability was derived. A general observation in that work was that increasing the diversity gain of the main channel can effectively reduce the secrecy outage probability. The single-input single-output multi-eavesdropper (SISOME) system was considered in [3], in which a single antenna transmitter communicates with a single antenna legitimate receiver in the presence of multiple eavesdroppers equipped with multiple antennas. In [4], transmit antenna selection (TAS) was proposed to provide secure communication. The proposed scheme consisted of a multiple antenna transmitter with a single radio frequency (RF) chain, a single antenna legitimate receiver, and a multiple antenna eavesdropper. In [5], cooperative jamming was introduced to confuse the eavesdropper in a multiple-input single-output (MISO) wiretap channel. By taking into account multiple antennas at the transceiver, the legitimate receiver, and the eavesdropper, the secrecy performance of several diversity combining schemes over independent and correlated fading

Manuscript received March 04, 2014; revised June 22, 2014; accepted September 14, 2014. The editor coordinating the review of this paper and approving it for publication was Prof. T. Charles Clancy.

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This research was supported in part by the U. S. National Science Foundation under Grant ECCS-1343210 and the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under Grant 102.04-2013.13.

channels was investigated in [6] and [7], respectively.

Unfortunately, exploiting multiple colocated antennas to secure the wireless transmission against eavesdropping and security attacks would often face the practical constraints of size and power, especially in small mobile and sensor terminals. One way around this is cooperative relaying to achieve spatial diversity using distributed terminals. Several dual-hop cooperative security schemes have been proposed and the impact of terminal cooperation on the secrecy rate was considered [8]–[16]. In particular, the performance of secure relay networks with different relaying protocols such as decode-and-forward (DF), amplify-and-forward (AF), and cooperative jamming was reported in [8], taking into account relay weights and power allocation. In [9] and [10], several secure selection schemes for opportunistic relaying were proposed. Relay selection and cooperative jamming was proposed in [11] and [12] for one-way relaying, and in [13] and [14] for two-way relaying. A new secrecy transmission protocol was proposed in [15], where the concept of interference alignment was combined with cooperative jamming to ensure that the artificial noise from the transmitters can be aligned at the destination, but not at the eavesdropper. The impact of cooperative jamming on MIMO wiretap channels was studied in [16].

It is important to note that although PHY layer security has been extensively studied in the open literature for both MIMO and cooperative communication networks, all previous works have assumed flat fading channels. In practice, multipath components are frequently present in wireless communication systems due to multiple reflectors, in which reflectors cause a time dispersion and frequency selective fading. If the signal bandwidth is larger than the frequency coherence bandwidth or the delay spread is larger than the symbol duration, the signal is distorted due to intersymbol interference (ISI). To avoid the use of equalizers in dealing with ISI, single carrier (SC) transmission is an alternative attractive solution which uses an increased symbol duration by forming a transmission block symbol [17], [18], with additional cyclic prefix (CP) symbols in front of the transmission block symbol. Thus, compared to orthogonal frequency-division multiplexing (OFDM) transmission, a block-wise processing is necessary for CP-SC transmission. There are several existing works and on-going activities in the context of CP-SC transmission in several different domains, including non-cooperative systems, cooperative relaying systems, and spectrum sharing systems, as follows.

- *Non-cooperative systems:* Opportunistic scheduling was proposed in [19] to achieve multiuser diversity. In [20] and [21], cyclic delay diversity (CDD) was employed for the frequency-domain equalizer (FDE), whereas distributed space-frequency block coding was employed with CP-SC systems [22] to achieve transmit diversity gain. Several channel estimators for CP-SC systems were investigated in [23]–[25].
- *Cooperative relaying systems:* For several relaying protocols such as DF and AF, as well as project and forward relaying [26], optimal power allocation [27], new receiver design [28], optimal training sequences for

channel estimation [29], and best terminal selection [30] were proposed to enhance the performance.

- *Spectrum sharing systems:* For cooperative spectrum sharing [31], [32], and non-cooperative spectrum sharing [33], CP-SC transmission was proposed considering the impact of multipath diversity on the system performance, taking into account several performance indicators such as outage probability, symbol error rate, and ergodic capacity.

While the aforementioned literature laid a solid foundation for the study of CP-SC systems, the PHY layer security issues with secrecy constraints in CP-SC transmission remain unknown. In this paper, to harness the aforementioned characteristics of multipath components in practice within the framework of PHY layer security, we focus on secure CP-SC transmission in DF relay networks. In contrast to the rich body of literature on PHY layer security, our main contributions are summarized as follows.

- Frequency selective fading is considered with constraints of PHY layer security, in which multiple relays and multiple destinations coexist with a cluster of eavesdroppers. A two-stage relay and destination selection is proposed to minimize the eavesdropping and maximize the signal power of the link between the relay and the destination.
- Analytical results for the secrecy outage probability, the probability of non-zero achievable secrecy rate, and the ergodic secrecy rate are derived in closed-form. The secrecy diversity gain and the secrecy array gain are calculated based on simplified expressions for the secrecy outage probability in the high SNR regime. Likewise, the multiplexing gain and the power cost are calculated based on simplified expressions for the ergodic secrecy rate in the high SNR regime.
- It is confirmed that the secrecy diversity gain is directly determined by the multipath diversity and the multiuser diversity between the relays and the destinations. The multiplexing gain is independent of the system and channel parameters including the number of multipaths, relays, eavesdroppers, and destinations. Our high SNR analysis shows that when the average received power at the eavesdropper is proportional to the counterpart at the destination, both the secrecy diversity gain and the secrecy capacity slope collapse to zero, thereby creating a secrecy outage floor and a secrecy capacity ceiling.

The rest of the paper is organized as follows. In Section II, we first detail the system and channel model of the proposed single carrier systems. In Section III, two-stage relay and destination selection is proposed under a group of eavesdroppers. Performance analysis of the considered physical system is presented in Section IV. Simulation results are presented in Section V and conclusions are drawn in Section VI.

*Notation:* The superscript  $(\cdot)^H$  denotes complex conjugate transposition;  $\mathbf{I}_N$  is an  $N \times N$  identity matrix;  $\mathbf{0}$  denotes an all-zeros matrix of appropriate dimensions;  $\mathcal{CN}(\mu, \sigma^2)$  denotes the complex Gaussian distribution with the mean  $\mu$  and the variance  $\sigma^2$ ;  $\mathbb{C}^{m \times n}$  denotes the vector space of all  $m \times n$  complex matrices;  $F_\varphi(\cdot)$  denotes the cumulative distribution

function (CDF) of the random variable (RV)  $\varphi$ ; and  $E_a\{\cdot\}$  denotes expectation with respect to  $a$ . The probability density function (PDF) of  $\varphi$  is denoted by  $f_\varphi(\cdot)$ ;  $[x]^+ = \max(x, 0)$  and  $\sum_{l_1, \dots, l_a}^i$  denotes a set of nonnegative integers  $\{l_1, \dots, l_a\}$  satisfying  $\sum_{t=1}^a l_t = i$ .

## II. SYSTEM AND CHANNEL MODEL

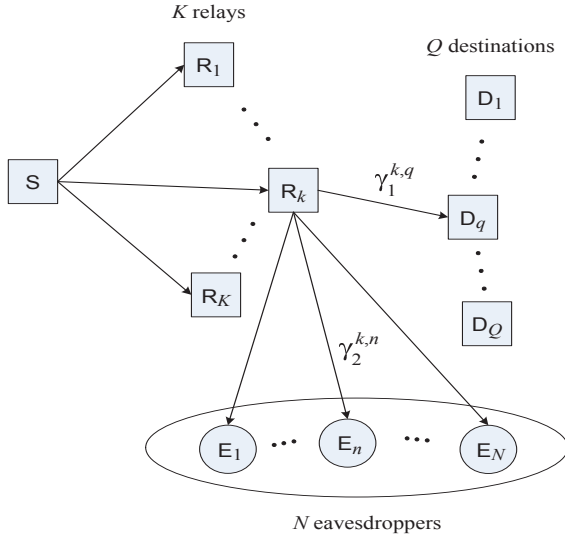


Fig. 1. PHY layer security for cooperative single carrier systems.

In the considered system, which is shown in Fig. 1, we assume the following set of instantaneous impulse channel responses.

- A set of channels  $\{\mathbf{g}^{k,q}, \forall k, q\}$  between a particular  $k$ th relay and the  $q$ th destination undergo a frequency selective fading. They are assumed to have the same  $N_1$  multipath components, i.e.,  $\mathbf{g}^{k,q} \triangleq [g_1^{k,q}, \dots, g_{N_1}^{k,q}]^T \in \mathbb{C}^{N_1 \times 1}$ , each of which is distributed by the complex white Gaussian distribution with the zero mean and the unit variance. The path losses over these channels are denoted by  $\{\alpha_1^{k,q}, \forall k, q\}$ .
- A set of channels  $\{\mathbf{h}^{k,1}, \dots, \mathbf{h}^{k,n}, \dots, \mathbf{h}^{k,N}\}$  between the  $k$ th relay and the  $N$  eavesdroppers undergo a frequency selective fading. They are assumed to have the same  $N_2$  multipath components, i.e.,  $\mathbf{h}^{k,n} \triangleq [h_1^{k,n}, \dots, h_{N_2}^{k,n}]^T \in \mathbb{C}^{N_2 \times 1}$ , each of which is distributed by the complex white Gaussian distribution with the zero mean and the unit variance. The path losses over these channels are denoted by  $\{\alpha_2^{k,n}, \forall k, n\}$ .
- The maximum channel length in the considered system is assumed to be  $N_g = \max(N_1, N_2, N_3)$ , where  $N_3$  denotes the multipath channel length between the source and relays.

For single-carrier cooperative transmission, we assume that

- Binary phase shift keying (BPSK) modulation is applied such that  $P$  modulated data symbols transmitted by the

source form a transmit symbol block  $\mathbf{x} \in \mathbb{C}^{P \times 1} \in \{-1, 1\}^P$  satisfying  $E_{\mathbf{x}}\{\mathbf{x}\} = \mathbf{0}$  and  $E_{\mathbf{x}}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_P$ .

- To prevent inter-block symbol interference (IBSI) [17], [27], [29], the CP comprising of  $P_g$  symbols is appended to the front of  $\mathbf{x}$ . It is also assume that  $P_g \geq N_g$ .
- We employ the selective-DF relaying protocol, which selects one relay and destination among their groups. This selection is accomplished via the proposed two-step selection scheme.
- We assume perfect decoding at each relay, so that error propagation does not exist in the considered system<sup>1</sup>.

The signal received at the  $n$ th eavesdropper from the  $k$ th relay is given by

$$\mathbf{r}^{k,n} = \sqrt{P_s \alpha_2^{k,n}} \mathbf{H}^{k,n} \mathbf{x} + \mathbf{n}_2^{k,n} \quad (1)$$

where  $P_s$  is the transmit power and  $\mathbf{H}^{k,n}$  is the right circulant matrix [27], [34] defined by  $\mathbf{h}^{k,n}$ . Also, we assume that  $\mathbf{n}_2^{k,n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_P)$ . Since we assume perfect decoding at all the relays and perfect knowledge of CSI<sup>2</sup>, channels between the source and the relays are not taken into account in (1) [10], [11].

*Definition 1:* Applying the properties of the right circulant channel matrix [27], [34], the instantaneous SNR between the  $k$ th relay and the  $n$ th eavesdropper is defined as

$$\gamma_2^{k,n} = \frac{P_s \alpha_2^{k,n} \|\mathbf{h}^{k,n}\|^2}{\sigma_n^2} = \tilde{\alpha}_2^{k,n} \|\mathbf{h}^{k,n}\|^2 \sim \chi^2(2N_2, \tilde{\alpha}_2^{k,n}) \quad (2)$$

where  $\tilde{\alpha}_2^{k,n} \triangleq \frac{P_s \alpha_2^{k,n}}{\sigma_n^2}$ , and the CDF and PDF of  $\gamma_2^{k,n}$  are, respectively, given by

$$F_{\gamma_2^{k,n}}(x) = 1 - e^{-x/\tilde{\alpha}_2^{k,n}} \sum_{l=0}^{N_2-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_2^{k,n}} \right)^l U(x) \quad \text{and} \\ f_{\gamma_2^{k,n}}(x) = \frac{1}{(\tilde{\alpha}_2^{k,n})^{N_2} (N_2 - 1)!} x^{N_2-1} e^{-x/\tilde{\alpha}_2^{k,n}} U(x) \quad (3)$$

where  $U(x)$  denotes the discrete unit function.

Now the received signal at the  $q$ th destination from the  $k$ th relay is given by

$$\mathbf{z}^{k,q} = \sqrt{P_s \alpha_1^{k,q}} \mathbf{G}^{k,q} \mathbf{x} + \mathbf{n}_1^{k,q} \quad (4)$$

where  $\mathbf{G}^{k,q}$  is the right circulant matrix defined by  $\mathbf{g}^{k,q}$ . Also, we assume that  $\mathbf{n}_1^{k,q} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_Q)$ . According to Definition 1, the instantaneous SNR of the link between the  $k$ th relay and the  $q$ th destination is given by  $\gamma_1^{k,q} = \frac{P_s \alpha_1^{k,q} \|\mathbf{g}^{k,q}\|^2}{\sigma_n^2} = \tilde{\alpha}_1^{k,q} \|\mathbf{g}^{k,q}\|^2 \sim \chi^2(2N_1, \tilde{\alpha}_1^{k,q})$ , so that the CDF of  $\gamma_1^{k,q}$  is given by

$$F_{\gamma_1^{k,q}}(x) = 1 - e^{-x/\tilde{\alpha}_1^{k,q}} \sum_{l=0}^{N_1-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_1^{k,q}} \right)^l U(x). \quad (5)$$

<sup>1</sup>Practically, the source and the relays are located in the same cluster yielding high received SNRs at the DF relays to successfully decode the messages.

<sup>2</sup>This assumption is commonly seen in the prior literature [8], [10]. The CSI of the eavesdropper channels can be obtained in the scenario where eavesdroppers are active.

In the sequel, we assume that pathloss components  $\alpha_2^{k,n}$  and  $\alpha_1^{k,q}$  are independent of the indices of the relay, eavesdropper, and destination, so that we have  $\alpha_2 = \{\alpha_2^{k,n}, \forall k, n\}$  and  $\alpha_1 = \{\alpha_1^{k,q}, \forall k, q\}$ .

### III. RELAY AND DESTINATION SELECTION UNDER A GROUP OF EAVESDROPPERS

In this section, we shall first propose the two-stage relay and destination selection procedure, in which a relay is selected to minimize the worst-case eavesdropping in the eavesdropper group, to decrease the amount of information that eavesdroppers wiretap. And then, the desired destination is selected from the chosen relay to have the maximum instantaneous SNR between them. That is, the relay and destination are chosen according to the following selection criteria:

$$\begin{aligned} \text{stage1} &: k^* = \min \arg_{k \in [1, K]} (\gamma_2^{k, \max}) \text{ and} \\ \text{stage2} &: q^* = \max \arg_{q \in [1, Q]} (\gamma_1^{k^*, q}) \end{aligned} \quad (6)$$

where  $\gamma_2^{k, \max}$  denotes the maximum instantaneous SNR among those of between the  $k$ th relay and  $N$  eavesdroppers. In addition,  $\gamma_1^{k^*, q}$  denotes the maximum instantaneous SNR between the selected relay and the  $q$ th destination. When  $Q = 1$ , the proposed relay and destination selection scheme becomes somewhat similar to that of [10] (Note that the relay selection based on maximal secrecy rate was analyzed in the prior literature such as [10], which brings large system overhead compared with our proposed scheme.). However, due to an achievable multiuser diversity, the proposed selection scheme will result in better secrecy outage probabilities, non-zero achievable secrecy rates, and ergodic secrecy rates. For this selection, we use a training symbol which has the same statistical properties as  $\mathbf{x}$ , and assume a quasi-stationary channel during its operation.

Next, the corresponding CDF and PDF for a link from a particular relay to a group of eavesdroppers will be derived. We start the derivation for the CDF of  $\gamma_2^{k, \max}$ , which is given by

$$F_{\gamma_2^{k, \max}}(x) = \left[ 1 - e^{-x/\tilde{\alpha}_2} \sum_{l=0}^{N_2-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_2} \right)^l \right]^N U(x) \quad (7)$$

where we assume that channels between a particular relay and  $N$  eavesdroppers are independent and identically distributed (i.i.d.).

Since  $\{\gamma_2^{1, \max}, \dots, \gamma_2^{K, \max}\}$  is a set of i.i.d. continuous random variables, the PDF of  $\gamma_2^{\min, \max} \triangleq \gamma_2^{k^*, \max} \triangleq \min(\gamma_2^{1, \max}, \dots, \gamma_2^{K, \max})$  can be derived in the following lemma.

**Lemma 1:** For the i.i.d. frequency selective fading channels between a particular relay and a group of eavesdroppers, the PDF of  $\gamma_2^{\min, \max}$  is given by (8) at the top of the next page.

*Proof:* A proof of this lemma is provided in Appendix A. ■

For the i.i.d. frequency selective fading channels between a particular relay and a group of  $Q$  destinations, the CDF of

$\gamma_1^{k^*, q^*} \triangleq \max(\gamma_1^{k^*, 1}, \dots, \gamma_1^{k^*, Q})$  is given by

$$F_{\gamma_1^{k^*, q^*}}(x) = \left[ 1 - e^{-x/\tilde{\alpha}_1} \sum_{l=0}^{N_1-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_1} \right)^l \right]^Q U(x). \quad (10)$$

### IV. PERFORMANCE ANALYSIS OF THE PHYSICAL SECRECY SYSTEM

The instantaneous secrecy rate is expressed as [6], [35]

$$C_s = \frac{1}{2} [\log_2(1 + \gamma_1^{k^*, q^*}) - \log_2(1 + \gamma_2^{\min, \max})]^+ \quad (11)$$

where  $\log_2(1 + \gamma_1^{k^*, q^*})$  is the instantaneous capacity of the channel between the chosen relay and the selected destination, whereas  $\log_2(1 + \gamma_2^{\min, \max})$  is the instantaneous capacity of the wiretap channel between the selected relay and the eavesdropper group. Having obtained PDFs and CDFs of SNRs achieved by the two-stage relay and destination selection scheme, the secrecy outage probability, the probability of non-zero achievable secrecy rate, and the ergodic secrecy rate will be derived. Then, an asymptotic analysis of the secrecy outage probability will be developed to see the asymptotic behavior of the system.

#### A. Secrecy Outage Probability

According to [7], the secrecy outage probability for a given secure rate,  $R$ , is given by

$$\begin{aligned} P_{\text{out}} &= \Pr(C_s < R) \\ &= \int_0^\infty F_{\gamma_1^{k^*, q^*}}(2^{2R}(1 + \gamma) - 1) f_{\gamma_2^{\min, \max}}(\gamma) d\gamma. \end{aligned} \quad (12)$$

A closed-form expression of (12) is provided by the following theorem.

**Theorem 1:** The secrecy outage probability of the single carrier system employing the proposed relay selection scheme in frequency selective fading is given by

$$\begin{aligned} P_{\text{out}} &= C \sum_{q=0}^{\infty} \sum_{q=0}^Q \binom{Q}{q} (-1)^q e^{-\frac{q(J_R-1)}{\tilde{\alpha}_1}} \\ &\quad \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\ &\quad \sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R - 1)^{\tilde{L}_1 - p} (J_R)^p \left( \frac{q J_R}{\tilde{\alpha}_1} + \beta_2 \right)^{-(p + \tilde{N}_2)} \\ &\quad (p + \tilde{N}_2 - 1)! \end{aligned} \quad (13)$$

where  $J_R \triangleq 2^{2R}$  and  $\tilde{L}_1 \triangleq \sum_{t=0}^{N_1-1} t w_{t+1}$ .

*Proof:* A detailed derivation is provided in Appendix B. ■

To explicitly see the secrecy diversity gain, we provide an asymptotic expression for (13) in the following theorem.

**Theorem 2:** The asymptotic secrecy outage probability at a fixed  $\tilde{\alpha}_2$  is given by

$$P_{\text{out}}^\infty \triangleq \lim_{\tilde{\alpha}_1 \rightarrow \infty} P_{\text{out}} = (G_a \tilde{\alpha}_1)^{-Q N_1} + O((\tilde{\alpha}_1)^{-Q N_1}) \quad (14)$$

$$\begin{aligned}
f_{\gamma_2^{\min, \max}}(x) &= \frac{KN}{(\tilde{\alpha}_2)^{N_2(N_2-1)!}} \sum_{k=0}^{K-1} \sum_{m=0}^{Nk} \sum_{j=0}^{N-1} \binom{K-1}{k} \binom{Nk}{m} \binom{N-1}{j} (-1)^{k+m+j} \\
&\quad \sum_{v_1, \dots, v_{N_2}}^m \sum_{u_1, \dots, u_{N_2}}^j \frac{m!}{v_1! \dots v_{N_2}!} \frac{j!}{u_1! \dots u_{N_2}!} \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{u_{t+1}}} \\
&\quad e^{-\frac{x(m+j+1)}{\tilde{\alpha}_2}} x^{N_2 + (\sum_{t=0}^{N_2-1} tv_{t+1}) + (\sum_{t=0}^{N_2-1} tu_{t+1}) - 1} \\
&= C \widetilde{\sum} e^{-\beta_2 x} x^{\tilde{N}_2 - 1} U(x)
\end{aligned} \tag{8}$$

where  $C \triangleq \frac{KN}{(\tilde{\alpha}_2)^{N_2(N_2-1)!}}$ ,  $\beta_2 \triangleq \frac{(m+j+1)}{\tilde{\alpha}_2}$ ,  $\tilde{N}_2 \triangleq N_2 + (\sum_{t=0}^{N_2-1} tv_{t+1}) + (\sum_{t=0}^{N_2-1} tu_{t+1})$ , and

$$\begin{aligned}
\widetilde{\sum} &\triangleq \sum_{k=0}^{K-1} \sum_{m=0}^{Nk} \sum_{j=0}^{N-1} \binom{K-1}{k} \binom{Nk}{m} \binom{N-1}{j} (-1)^{k+m+j} \\
&\quad \sum_{v_1, \dots, v_{N_2}}^m \sum_{u_1, \dots, u_{N_2}}^j \frac{m!}{v_1! \dots v_{N_2}!} \frac{j!}{u_1! \dots u_{N_2}!} \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{u_{t+1}}}.
\end{aligned} \tag{9}$$

where the secrecy array gain is given by

$$\begin{aligned}
G_a &= \left[ \frac{\hat{C}}{(N_1!)^Q} \widetilde{\sum} \sum_{l=0}^{Q N_1} \binom{Q N_1}{l} (J_R - 1)^{Q N_1 - l} \right. \\
&\quad \left. (J_R)^l (\tilde{\alpha}_2)^l \frac{(l + \tilde{N}_2 - 1)!}{(\hat{\beta})^{l + \tilde{N}_2}} \right]^{-\frac{1}{Q N_1}}
\end{aligned} \tag{15}$$

with  $\hat{C} \triangleq \frac{KN}{(N_2-1)!}$ ,  $\hat{\beta} \triangleq m + j + 1$ , and  $\widetilde{\sum}$ , which is given by

$$\begin{aligned}
\widetilde{\sum} &\triangleq \sum_{k=0}^{K-1} \sum_{m=0}^{Nk} \sum_{j=0}^{N-1} \binom{K-1}{k} \binom{Nk}{m} \binom{N-1}{j} (-1)^{k+m+j} \\
&\quad \sum_{v_1, \dots, v_{N_2}}^m \sum_{u_1, \dots, u_{N_2}}^j \frac{m!}{v_1! \dots v_{N_2}!} \frac{j!}{u_1! \dots u_{N_2}!} \\
&\quad \frac{1}{\prod_{t=0}^{N_2-1} (t!)^{v_{t+1}}} \frac{1}{\prod_{t=0}^{N_2-1} (t!)^{u_{t+1}}}.
\end{aligned} \tag{16}$$

*Proof:* A detailed proof of this theorem is provided in Appendix C. ■

This theorem shows that the secrecy diversity gain is  $Q N_1$ , which is the product of the multipath diversity gain and the multiuser diversity gain achievable between the selected relay and the  $Q$  destinations.

*Corollary 1:* When  $\tilde{\alpha}_1 \rightarrow \infty$ ,  $\tilde{\alpha}_2 \rightarrow \infty$  with  $\frac{\tilde{\alpha}_1}{\tilde{\alpha}_2} = \kappa$ , then the asymptotic secrecy outage probability is given by

$$P_{\text{out}}^\infty = \frac{\hat{C}}{(N_1!)^Q} \widetilde{\sum} (\kappa)^{Q N_1} (J_R)^{Q N_1} \frac{(Q N_1 + \tilde{N}_2 - 1)!}{(\hat{\beta})^{Q N_1 + \tilde{N}_2}} \tag{17}$$

which shows that the secrecy diversity gain is not achievable for this particular case.

### B. The probability of non-zero achievable secrecy rate

In the following, we shall derive the probability of non-zero achievable secrecy rate.

*Corollary 2:* The probability of non-zero achievable secrecy rate is provided by (18) at the top of the next page. In (18), we have defined  $\tilde{N}_1 \triangleq N_1 + (\sum_{t=0}^{N_1-1} tw_{t+1}) + (\sum_{t=0}^{N_2-1} tv_{t+1})$ .

*Proof:* A proof of this corollary is provided in Appendix D. ■

To investigate the effect of the diversity gain on the convergence behavior of the probability of non-zero achievable secrecy rate to  $Pr(C_s > 0) = 1$ , we derive an asymptotic probability of non-zero achievable secrecy rate. According to (D.3), the probability of non-zero achievable secrecy rate can be rewritten as

$$Pr(C_s > 0) = 1 - \int_0^\infty F_{\gamma_1^{k^*, q^*}}(x) f_{\gamma_2^{\min, \max}}(x) dx. \tag{19}$$

Substituting (C.1) and (8) into (19), we get the following asymptotic probability of non-zero achievable secrecy rate

$$Pr(C_s^\infty > 0) = 1 - \frac{C}{(N_1!)^Q} \left( \frac{1}{\tilde{\alpha}_1} \right)^{N_1 Q} \widetilde{\sum} \frac{(N_1 Q + \tilde{N}_2 - 1)!}{(\beta_2)^{N_1 Q + \tilde{N}_2}} \tag{20}$$

which shows that the multipath diversity gain and the multiuser diversity gain simultaneously affect the convergence speed of the non-zero achievable secrecy rate to  $Pr(C_s > 0) = 1$ . In the following, we shall derive the ergodic secrecy rate for the proposed system.

### C. Ergodic Secrecy Rate

The ergodic secrecy rate is defined as the instantaneous secrecy rate  $C_s$  averaged over  $\gamma_1^{j^*, q^*}$  and  $\gamma_2^{\min, \max}$ . As such, we formulate the ergodic secrecy rate as

$$\bar{C}_s = \int_0^\infty \int_0^\infty C_s f_{\gamma_1^{j^*, q^*}}(x_1) f_{\gamma_2^{\min, \max}}(x_2) dx_1 dx_2. \tag{21}$$

Substituting (11) into (21), and applying some algebraic manipulations, we obtain

$$\bar{C}_s = \frac{1}{2 \log(2)} \int_0^\infty \frac{F_{\gamma_2^{\min, \max}}(x_2)}{1 + x_2} \left( 1 - F_{\gamma_1^{j^*, q^*}}(x_2) \right) dx_2. \tag{22}$$

$$Pr(C_s > 0) = 1 - \frac{Q}{(\tilde{\alpha}_1)^{N_1}(N_1 - 1)!} \sum_{k=0}^K \sum_{m=0}^{N_k} \sum_{q=0}^{Q-1} \binom{Q-1}{q} \binom{K}{k} \binom{N_k}{m} (-1)^{q+k+m} \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \left( \frac{m}{\tilde{\alpha}_2} + \frac{q+1}{\tilde{\alpha}_1} \right)^{-\tilde{N}_1} (\tilde{N}_1 - 1)!. \quad (18)$$

Based on the PDF of  $\gamma_2^{\min, \max}$  given in (8), the CDF of  $\gamma_2^{\min, \max}$  is given by

$$F_{\gamma_2^{\min, \max}}(x) = \int_0^x f_{\gamma_2^{\min, \max}}(t) dt = C \sum_{n_1=0}^{\tilde{N}_2-1} \left[ \frac{(\tilde{N}_2 - 1)!}{(\beta_2)^{\tilde{N}_2}} - e^{-\beta_2 x} \sum_{n_1=0}^{\tilde{N}_2-1} \frac{(\tilde{N}_2 - 1)!}{n_1!} \frac{x^{n_1}}{(\beta_2)^{\tilde{N}_2 - n_1}} \right]. \quad (23)$$

In addition, by employing binomial and multinomial formulas, the CDF of  $\gamma_1^{k^*, q^*}$  in (10) can be re-expressed as

$$F_{\gamma_1^{k^*, q^*}}(x) = 1 + \sum_{q=1}^Q \binom{Q}{q} (-1)^q e^{-qx/\tilde{\alpha}_1} \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{x^{\tilde{L}_1}}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}}. \quad (24)$$

Substituting (23) and (24) into (22), and using the confluent hypergeometric function [36, eq. (9.211.4)] given by  $\Psi(\alpha, \gamma; z) = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-zt} t^{\alpha-1} (1+t)^{\gamma-\alpha-1} dt$ , we obtain the ergodic secrecy rate expressed in (25) at the top of the next page.

In order to gather further insight, we present the asymptotic ergodic secrecy rate. We first consider the case of  $\tilde{\alpha}_1 \rightarrow \infty$  and a fixed  $\tilde{\alpha}_2$ , and provide the following corollary.

*Corollary 3:* The asymptotic ergodic secrecy rate at  $\tilde{\alpha}_1 \rightarrow \infty$  and a fixed  $\tilde{\alpha}_2$  is given by (26) at the top of the next page. In (26),  $\psi(\cdot)$  is the digamma function [37].

*Proof:* A proof of this corollary is provided in Appendix E. ■

With the help of (26), we confirm that the multiplexing gain [38] is 1/2 in bits/sec/Hz/(3 dB), which is given by

$$S^\infty = \lim_{\tilde{\alpha}_1 \rightarrow \infty} \frac{\bar{C}_1^\infty}{\log_2(\tilde{\alpha}_1)} = \frac{1}{2}. \quad (27)$$

It is indicated from (27) that under these circumstances, secure communication achieves the same spectral efficiency as communication without eavesdropping. Moreover, using (26), we can easily calculate the additional power cost for different network parameters while maintaining a specified target ergodic secrecy rate. For example, we consider different numbers of relays  $K_1$  and  $K_2$  with  $K_1 > K_2$ . Compared to the  $K_1$  case, the additional power cost in achieving the specified target ergodic secrecy rate in the  $K_2$  scenario is calculated as

$$\Delta P \text{ (dB)} = \frac{10}{\log 10} [\eta(K_1) - \eta(K_2)] \quad (28)$$

where

$$\eta(K) = \sum_{k=1}^K \sum_{m=1}^{N_k} \binom{K}{k} \binom{N_k}{m} (-1)^{k+m+1} \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \frac{\Gamma(\sum_{t=0}^{N_2-1} tv_{t+1} + 1)}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \Psi\left(\sum_{t=0}^{N_2-1} tv_{t+1} + 1, \sum_{t=0}^{N_2-1} tv_{t+1} + 1; m/\tilde{\alpha}_2\right).$$

Similarly, the additional power cost in achieving the specified target ergodic secrecy rate under different numbers of destinations or eavesdroppers can be accordingly obtained.

We next consider the case of  $\tilde{\alpha}_1 \rightarrow \infty$  and  $\tilde{\alpha}_2 \rightarrow \infty$  with  $\frac{\tilde{\alpha}_1}{\tilde{\alpha}_2} = \kappa$ , and provide the following corollary.

*Corollary 4:* The asymptotic ergodic secrecy rate at  $\tilde{\alpha}_1 \rightarrow \infty$  and  $\tilde{\alpha}_2 \rightarrow \infty$  with  $\frac{\tilde{\alpha}_1}{\tilde{\alpha}_2} = \kappa$  is given by (29) at the top of the next page.

*Proof:* A proof of this corollary is provided in Appendix F. ■

It is indicated from (29) that a capacity ceiling exists in this case.

#### D. The Effects of Multiple Antennas at the Eavesdroppers

We shall investigate the effect of multiple antennas at the eavesdroppers. Using MRC at each eavesdropper, the received signal expressed in (1) becomes

$$\mathbf{r}^{k,n} = \sqrt{P_s \alpha_2^{k,n}} \sum_{r=1}^M (\tilde{\mathbf{H}}_r^{k,n})^H \mathbf{H}_r^{k,n} \mathbf{x} + \sum_{r=1}^M (\tilde{\mathbf{H}}_r^{k,n})^H \mathbf{n}_1^{k,n} \quad (30)$$

where  $\mathbf{H}_r^{k,n}$  is the right circulant matrix formed for a link from the  $k$ th relay to the  $r$ th receive antenna branch at the  $n$ th eavesdropper. In the formulation of (30), we assume  $M$  antennas at the each eavesdropper, and  $\alpha_2^{k,n}$  is independent of the antenna branches. In addition,  $\tilde{\mathbf{H}}_r^{k,n}$  is the receive matrix for the  $r$ th receive antenna branch at the  $n$ th eavesdropper. The maximum instantaneous post-processing SNR due to MRC, which is imposed  $\tilde{\mathbf{H}}_r^{k,n} = \mathbf{H}_r^{k,n}$ , becomes [33]

$$\gamma_2^{k,n,\text{eMRC}} = \frac{P_s \alpha_2^{k,n} \sum_{r=1}^M \|\mathbf{h}_r^{k,n}\|^2}{\sigma_n^2}. \quad (31)$$

Comparing to the expression in (2), we can easily see that

$$\gamma_2^{k,n,\text{eMRC}} = \tilde{\alpha}_2^{k,n} \sum_{r=1}^M \|\mathbf{h}_r^{k,n}\|^2 \sim \chi^2(2N_2 M, \tilde{\alpha}_2^{k,n}). \quad (32)$$

$$\begin{aligned}\bar{C}_s = & -\frac{C}{2\log(2)} \widetilde{\sum_{q=1}^Q} \sum_{w_1, \dots, w_{N_1}} \binom{Q}{q} (-1)^q \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t!(\tilde{\alpha}_1)^t)^{w_{t+1}}} \\ & \left( \frac{\Gamma(\tilde{N}_2)\Gamma(\tilde{L}_1+1)}{(\beta_2)^{\tilde{N}_2}} \Psi(\tilde{L}_1+1, \tilde{L}_1+1; q/\tilde{\alpha}_1) - \sum_{n_1=0}^{\tilde{N}_2-1} \frac{\Gamma(\tilde{N}_2)\Gamma(\tilde{L}_1+n_1+1)}{n_1!(\beta_2)^{\tilde{N}_2-n_1}} \right. \\ & \left. \Psi(\tilde{L}_1+n_1+1, \tilde{L}_1+n_1+1; \beta_2+q/\tilde{\alpha}_1) \right). \quad (25)\end{aligned}$$


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$$\begin{aligned}\bar{C}_1^\infty = & \frac{1}{2}\log_2(\tilde{\alpha}_1) + \frac{1}{2\log(2)} \left[ \frac{Q}{(N_1-1)!} \sum_{q=0}^{Q-1} \binom{Q-1}{q} (-1)^q \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{N_1-1} (t!)^{w_{t+1}}} \right. \\ & \frac{\Gamma(N_1+\tilde{L}_1)}{(q+1)^{N_1+\tilde{L}_1}} [\psi(N_1+\tilde{L}_1) - \log(q+1)] + \sum_{k=1}^K \sum_{m=1}^{Nk} \binom{K}{k} \binom{Nk}{m} (-1)^{k+m+1} \\ & \left. \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \frac{\Gamma(\sum_{t=0}^{N_2-1} tv_{t+1}+1)}{\prod_{t=0}^{N_2-1} (t!(\tilde{\alpha}_2)^t)^{v_{t+1}}} \Psi(\sum_{t=0}^{N_2-1} tv_{t+1}+1, \sum_{t=0}^{N_2-1} tv_{t+1}+1; m/\tilde{\alpha}_2) \right]. \quad (26)\end{aligned}$$


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$$\begin{aligned}\bar{C}_2^\infty = & \frac{1}{2}\log_2(\kappa) + \frac{1}{2\log(2)} \left[ \frac{Q}{(N_1-1)!} \sum_{q=0}^{Q-1} \binom{Q-1}{q} (-1)^q \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{N_1-1} (t!)^{w_{t+1}}} \right. \\ & \left. \frac{\Gamma(N_1+\tilde{L}_1)}{(q+1)^{N_1+\tilde{L}_1}} [\psi(N_1+\tilde{L}_1) - \log(q+1)] - \hat{C} \sum \frac{\Gamma(\tilde{N}_2)}{(\hat{\beta})^{\tilde{N}_2}} [\psi(\tilde{N}_2) - \log(\hat{\beta})] \right]. \quad (29)\end{aligned}$$


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$$\begin{aligned}P_{\text{out}}^{\text{eMRC}} = & C^{\text{eMRC}} \widetilde{\sum}^{\text{eMRC}} \sum_{q=0}^Q \binom{Q}{q} (-1)^q e^{-\frac{q(J_R-1)}{\tilde{\alpha}_1}} \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t!(\tilde{\alpha}_1)^t)^{w_{t+1}}} \\ & \sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R-1)^{\tilde{L}_1-p} (J_R)^p \left( \frac{qJ_R}{\tilde{\alpha}_1} + \beta_2 \right)^{-(p+\tilde{N}_2^{\text{eMRC}})} (p+\tilde{N}_2^{\text{eMRC}}-1)!, \\ Pr(C_s^{\text{eMRC}} > 0) = & 1 - \frac{Q}{(\tilde{\alpha}_1)^{N_1} (N_1-1)!} \sum_{k=0}^K \sum_{m=0}^{Nk} \sum_{q=0}^{Q-1} \binom{Q-1}{q} \binom{K}{k} \binom{Nk}{m} (-1)^{q+k+m} \\ & \sum_{v_1, \dots, v_{MN_2}}^m \left( \frac{m!}{v_1! \dots v_{MN_2}!} \right) \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{MN_2-1} (t!(\tilde{\alpha}_2)^t)^{v_{t+1}}} \\ & \frac{1}{\prod_{t=0}^{N_1-1} (t!(\tilde{\alpha}_1)^t)^{w_{t+1}}} \left( \frac{m}{\tilde{\alpha}_2} + \frac{q+1}{\tilde{\alpha}_1} \right)^{-\tilde{N}_1} (\tilde{N}_1-1)!, \text{ and} \\ \bar{C}_s^{\text{eMRC}} = & -\frac{1}{2\log(2)} C^{\text{eMRC}} \widetilde{\sum}^{\text{eMRC}} \sum_{q=1}^Q \binom{Q}{q} (-1)^q \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t!(\tilde{\alpha}_1)^t)^{w_{t+1}}} \\ & \left[ \frac{\Gamma(\tilde{N}_2^{\text{eMRC}})\Gamma(\tilde{L}_1+1)}{(\beta_2)^{\tilde{N}_2^{\text{eMRC}}}} \Psi(\tilde{L}_1+1, \tilde{L}_1+1; q/\tilde{\alpha}_1) - \sum_{n_1=0}^{\tilde{N}_2^{\text{eMRC}}-1} \frac{\Gamma(\tilde{N}_2^{\text{eMRC}})\Gamma(\tilde{L}_1+n_1+1)}{n_1!(\beta_2)^{\tilde{N}_2^{\text{eMRC}}-n_1}} \right. \\ & \left. \Psi(\tilde{L}_1+n_1+1, \tilde{L}_1+n_1+1; \beta_2+q/\tilde{\alpha}_1) \right]. \quad (33)\end{aligned}$$


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Using the statistical properties of  $\gamma_2^{k,n,\text{eMRC}}$ , the performance metrics, such as the secrecy outage probability, the probability of non-zero achievable secrecy rate, and the ergodic secrecy rate can be derived. Their corresponding expressions are given by (33) at the bottom of the previous page. In (33), we have defined  $C^{\text{eMRC}} \triangleq C|_{N_2 \rightarrow MN_2}$ ,  $\widetilde{\Sigma}^{\text{eMRC}} \triangleq \widetilde{\Sigma}|_{N_2 \rightarrow MN_2}$ , and  $\widetilde{N}_2^{\text{eMRC}} \triangleq MN_2 + (\sum_{t=0}^{MN_2-1} tv_{t+1}) + (\sum_{t=0}^{MN_2-1} tu_{t+1})$ .

*Corollary 5:* The multiple antennas employed in the form of MRC at each eavesdropper do not influence the secrecy diversity gain. They can only change the secrecy array gain.

*Proof:* According to Theorem 2, the asymptotic secrecy outage probability at a fixed  $\tilde{\alpha}_2$  is given by

$$P_{\text{out}}^{\infty, \text{eMRC}} = (G_a^{\text{eMRC}} \tilde{\alpha}_1)^{-Q N_1} + O((\tilde{\alpha}_1)^{-Q N_1}) \quad (34)$$

where

$$G_a^{\text{eMRC}} = \left[ \frac{\hat{C}^{\text{eMRC}}}{(N_1!)^Q} \sum_{l=0}^{\widetilde{\Sigma}^{\text{eMRC}}} \sum_{l=0}^{Q N_1} \binom{Q N_1}{l} (J_R - 1)^{Q N_1 - l} (J_R)^l (\tilde{\alpha}_2)^l \frac{(l + \widetilde{N}_2^{\text{eMRC}} - 1)!}{(\hat{\beta})^{l + \widetilde{N}_2^{\text{eMRC}}} \right]^{-\frac{1}{Q N_1}} \quad (35)$$

with  $\hat{C}^{\text{eMRC}} \triangleq \hat{C}|_{N_2 \rightarrow MN_2}$  and  $\widetilde{\Sigma}^{\text{eMRC}} \triangleq \widetilde{\Sigma}|_{N_2 \rightarrow MN_2}$ , where  $\hat{C}$  and  $\widetilde{\Sigma}$  are specified in (16). From (34), we can readily see that MRC at the each eavesdropper does not affect the secrecy diversity gain. ■

*Corollary 6:* The multiple antennas employed in the form of MRC at the eavesdroppers do not influence the multiplexing gain. They can only change the additional power cost for a specified target ergodic secrecy rate.

*Proof:* According to Corollary 3, the asymptotic ergodic secrecy rate at a fixed  $\tilde{\alpha}_2$  is given by only interchanging the parameter  $N_2 \rightarrow MN_2$ . From (27), we see that the multiplexing gain is still 1/2, and MRC at the eavesdroppers impacts the additional power cost as shown in (28). ■

## V. SIMULATION RESULTS

For the simulations, we use BPSK modulation. The transmission block size is formed by 64 BPSK symbols. The CP length is given by 16 BPSK symbols. Every channel vectors are generated by  $\mathbf{h}^{k,n} \sim CN(\mathbf{0}, \mathbf{I}_{N_2})$ ,  $\forall k, n$  and  $\mathbf{g}^{k,q} \sim CN(\mathbf{0}, \mathbf{I}_{N_1})$ ,  $\forall k, q$ . The curves obtained via actual link simulations are denoted by **Ex**, whereas analytically derived curves are denoted by **An**. Asymptotically obtained curves are denoted by **As** in the following figures.

### A. Secrecy Outage Probability

Figs. 2-4 show the secrecy outage probability for various scenarios. Fig. 2 shows the secrecy outage probability for various values of  $N_1$  at fixed values of  $(K = 4, N = 2, N_2 = 3, Q = 1, M = 1, R = 1)$  and  $\tilde{\alpha}_2 = 5$  dB. As Theorem 2 proves, a lower secrecy outage probability is achieved by a bigger value of  $N_1$ . In this particular scenario, the secrecy diversity gain becomes  $N_1$ . We can see exact matches between the analytically derived curves and the simulation obtained curves for the outage probability. Fig. 3 shows the secrecy

outage probability for various values of  $Q$  and  $M$  at fixed value of  $(K = 4, N = 2, N_1 = 3, N_2 = 2, R = 1)$  and  $\tilde{\alpha}_2 = 5$  dB. We can observe the effect of the multiuser diversity gain on the secrecy outage probability. As  $Q$  increases, a lower secrecy outage probability is obtained due to the multiuser diversity. We can also observe the effect of multiple antennas at the eavesdroppers. For the same channel length and the number of destinations, for example,  $(N_1 = 3, N_2 = 2, Q = 1, M = 1)$  has a 3 dB gain over  $(N_1 = 3, N_2 = 2, Q = 1, M = 2)$  at  $1 \times 10^{-3}$  outage probability. Similar behavior can be observed as  $M$  becomes larger. Moreover, it can be seen that  $N$ , the number of eavesdroppers, does not change the secrecy diversity gain. Fig. 4 verifies the derived asymptotic secrecy outage probability at a fixed  $\tilde{\alpha}_2$ . As  $\tilde{\alpha}_1$  increases, the asymptotic curves approaches the simulation obtained curves for various values of  $N_1$ ,  $Q$ , and  $M$ . From these curves, we can see that the secrecy diversity gain is  $N_1 Q$ , which is determined by the multipath diversity gain,  $N_1$ , and the multiuser diversity gain,  $Q$ . It is irrespective of  $M$ . A similar overall diversity gain is obtained in [27], which does not consider eavesdroppers.

### B. The Non-Zero achievable Secrecy Rate

Fig. 5 illustrates the non-zero achievable secrecy rate for various values of  $N_1$ ,  $M$ , and  $Q$ . At fixed  $(K = 4, N = 2)$  and  $\tilde{\alpha}_2 = 5$  dB, this figure shows that  $(N_1 = 2, M = 2, Q = 1)$  has the slowest convergence speed arriving at  $Pr(C_{\min} > 0) = 0.999$  due to the smallest achievable diversity gain and the value of  $M$ . Although  $(N_1 = 2, M = 2, Q = 1)$  has the same diversity gain as  $(N_1 = 2, M = 1, Q = 1)$ , its convergence speed is slowest due to greater eavesdropping capability of eavesdroppers. If we compare two particular scenarios, such as  $(N_1 = 2, M = 2, Q = 1)$  and  $(N_1 = 3, M = 2, Q = 1)$ , then the multipath diversity is seen to be one of the key factor in determining the convergence speed, whereas by comparing  $(N_1 = 2, M = 2, Q = 1)$  with  $(N_1 = 2, M = 2, Q = 2)$ , we can see that the multiuser diversity is another key factor in

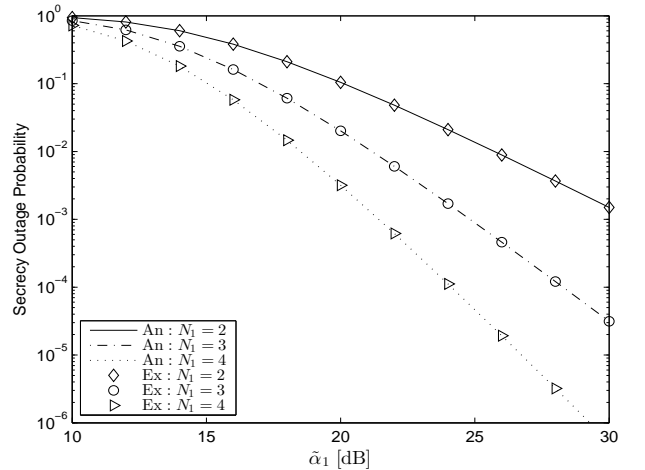


Fig. 2. Secrecy outage probability for various values of  $N_1$  at fixed values of  $(N_2 = 3, R = 1)$  and  $\tilde{\alpha}_2 = 5$  dB.

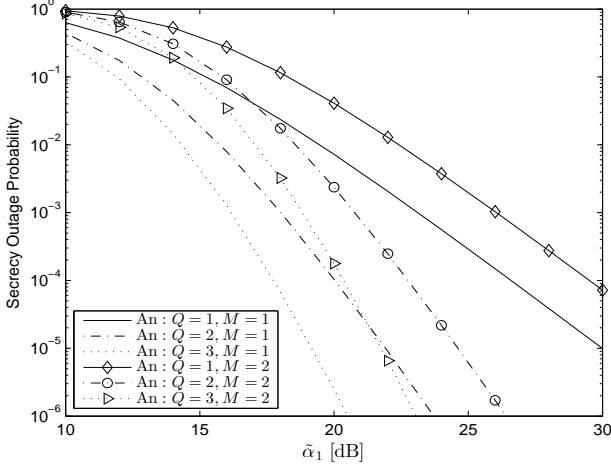


Fig. 3. Secrecy outage probability for various values of  $Q$  and  $M$  at fixed values of  $(N_1 = 3, N_2 = 2, R = 1)$  and  $\tilde{\alpha}_2 = 5$  dB.

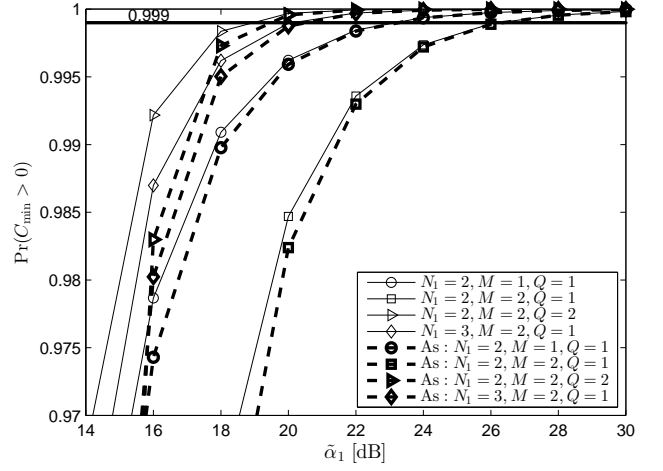


Fig. 5. Non-zero achievable secrecy rate for various values of  $N_1$ ,  $M$ , and  $Q$  at fixed values of  $N_2 = 2$  and  $\tilde{\alpha}_2 = 5$  dB.

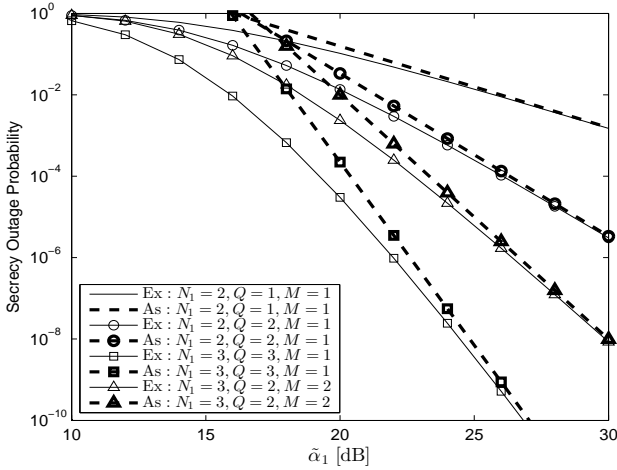


Fig. 4. Asymptotic secrecy outage probability for various values of  $N_1$ ,  $Q$ , and  $M$  at fixed values of  $(N_2 = 3, R = 1)$  and  $\tilde{\alpha}_2 = 5$  dB.

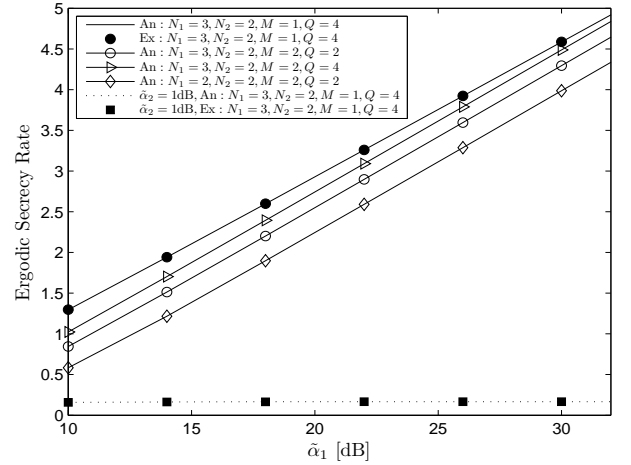


Fig. 6. Ergodic secrecy rate for various values of  $(K, N_1, N_2, M, Q)$ .

determining the convergence speed of the non-zero achievable secrecy rate.

### C. The Ergodic Secrecy Rate

In Fig. 6, we first compare the derived ergodic secrecy rate with the simulation obtained ergodic secrecy rate for the case of  $(N_1 = 3, N_2 = 2, M = 1, Q = 4)$ . We assume a fixed number of eavesdroppers ( $N = 3$ ) and a single relay ( $K = 1$ ). Perfect matchings between them can be observed. From this figure, we can compare several scenarios to investigate the effects from the system configurations and channels.

- The effect of eavesdropping: More eavesdropping reduces the ergodic secrecy rate. For example,  $(N_1 = 3, N_2 = 2, M = 2, Q = 4)$  vs.  $(N_1 = 3, N_2 = 2, M = 1, Q = 4)$ .
- The effect of multipath diversity which is achievable between the relay and the destination: Higher multipath diversity gain results in a higher ergodic secrecy rate.

For example,  $(N_1 = 3, N_2 = 2, M = 2, Q = 2)$  vs.  $(N_1 = 2, N_2 = 2, M = 2, Q = 2)$ .

- The effect of number of destinations: With more destinations, a higher ergodic secrecy rate can be obtained due to a larger multiuser diversity gain. For example,  $(N_1 = 2, N_2 = 2, M = 2, Q = 4)$  vs.  $(N_1 = 2, N_2 = 2, M = 2, Q = 2)$ .
- The effect of fixed  $\tilde{\alpha}_2$ : As Corollary 4 verified, capacity ceilings are intrinsic for this case.

In Fig. 7, we show the asymptotic ergodic secrecy rate for various values of  $(K, N_1, N_2, M, Q)$  at a fixed number of eavesdroppers  $N = 3$  and  $\tilde{\alpha}_2$ . This plot shows the corresponding asymptotic ergodic secrecy rate obtained from Corollary 3. As  $\tilde{\alpha}_1$  increases, the differences between the analytical ergodic secrecy rates and the asymptotic ergodic secrecy rates are negligible. We can also easily see that the multipath diversity and the multiuser diversity are two key factors in determining the ergodic secrecy rates. According to (28), a total of five

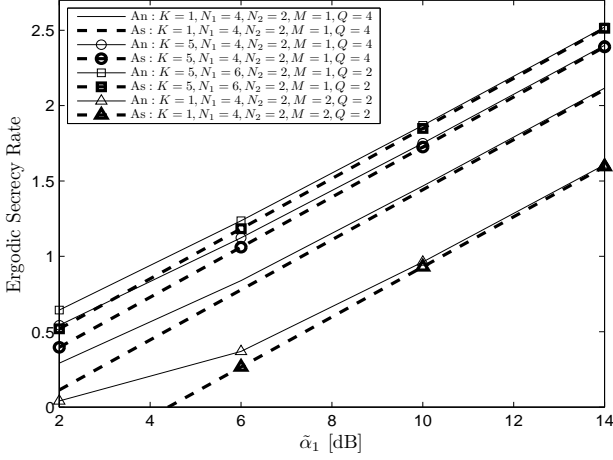


Fig. 7. Ergodic secrecy rate for various values of  $N_1$  and  $Q$  at fixed values of  $(K=4, N=2)$  and  $\tilde{\alpha}_2 = 1$  dB.

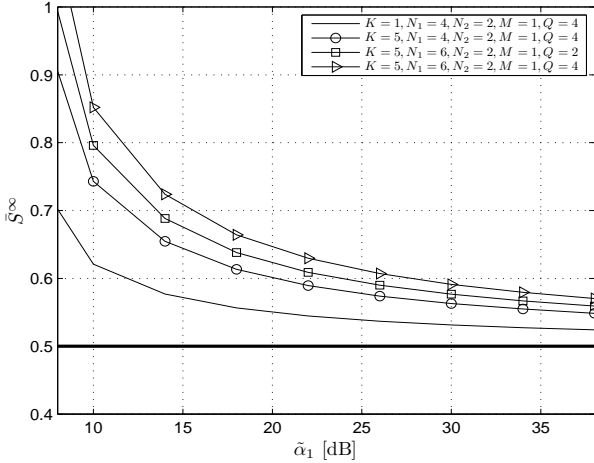


Fig. 8. Multiplexing gain  $S^\infty$ .

relays can reduce 0.8 dB power than a single relay in achieving 2.0 secrecy rate. Fig. 8 shows the multiplexing gain  $S^\infty$  as a function of  $(K, N_1, Q)$ , which are the key system and channel parameters in determining the diversity gain. As  $\tilde{\alpha}_1$  increases, the multiplexing gain  $S^\infty$  approaches  $1/2$ . Since a larger diversity has a more influence from the second term in the right hand side of (26), the convergence speed to  $1/2$  becomes slower as the diversity gain increases.

## VI. CONCLUSIONS

In this paper, we have proposed cooperative single carrier systems with multiple relays and destinations. A coexisting group of eavesdroppers have been assumed to eavesdrop the relays. For this challenging environment, we have proposed a two-stage relay and destination selection scheme: 1) relay is selected to minimize the worst-case eavesdropping, and 2) the desired destination is selected to achieve the multiuser diversity gain. We have derived the secrecy outage probability,

the non-zero secrecy rate, and the ergodic secrecy rate. From the derivations and the link simulations, the diversity gain has been shown to be determined by the multipath diversity gain and the multiuser diversity gain. Having derived the asymptotic ergodic secrecy rate, the multiplexing gain has been shown to be equal to the number of hops.

## APPENDIX A: A DETAILED DERIVATION OF LEMMA 1

According to the order statistics, the PDF of  $\gamma_2^{\min, \max}$  is given by

$$f_{\gamma_2^{\min, \max}}(x) = K(1 - F_{\gamma_2^{\max}}(x))^{K-1} f_{\gamma_2^{\max}}(x). \quad (\text{A.1})$$

Binomial and multinomial formulas provide the following expression for  $f_{\gamma_2^{\max}}(x)$ :

$$f_{\gamma_2^{\max}}(x) = \frac{N}{(\tilde{\alpha}_2)^{N_2} (N_2 - 1)!} \sum_{j=0}^{N-1} \binom{N-1}{j} (-1)^j e^{-\frac{x(j+1)}{\tilde{\alpha}_2}} \sum_{u_1, \dots, u_{N_2}}^j \left( \frac{j!}{u_1! \dots u_{N_2}!} \right) \frac{x^{N_2 + \sum_{t=0}^{N_2-1} t u_{t+1} - 1}}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{u_{t+1}}}. \quad (\text{A.2})$$

Again binomial and multinomial formulas lead us to get the following expression for  $(1 - F_{\gamma_2^{\max}}(x))^{K-1}$ :

$$\begin{aligned} & (1 - F_{\gamma_2^{\max}}(x))^{K-1} \\ &= \left[ 1 - \left( 1 - e^{-x/\tilde{\alpha}_2} \sum_{l=0}^{N_2-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_2} \right)^l \right)^N \right]^{K-1} \\ &= \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \left( 1 - e^{-x/\tilde{\alpha}_2} \sum_{l=0}^{N_2-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_2} \right)^l \right)^{kN} \\ &= \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \sum_{m=0}^{Nk} \binom{Nk}{m} (-1)^m e^{-mx/\tilde{\alpha}_2} \\ & \quad \left( \sum_{l=0}^{N_2-1} \frac{1}{l!} \left( \frac{x}{\tilde{\alpha}_2} \right)^l \right)^m \\ &= \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \sum_{m=0}^{Nk} \binom{Nk}{m} (-1)^m e^{-mx/\tilde{\alpha}_2} \\ & \quad \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \frac{x^{\sum_{t=0}^{N_2-1} t v_{t+1}}}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}}. \quad (\text{A.3}) \end{aligned}$$

Multiplying (A.2) and (A.3) and after some manipulations, yields (8).

## APPENDIX B: A DETAILED DERIVATION OF THEOREM 1

Now substituting  $f_{\gamma_2^{\min, \max}}(\gamma)$ , which is derived in (8) and  $F_{\gamma_1^{k^*, q^*}}(\gamma)$ , which is derived in (5) into (12), we have (B.1) at the top of the next page. Using multinomial and binomial formulas,  $J_1$  becomes

$$J_1 = \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R - 1)^{\tilde{L}_1 - p} (J_R)^p \gamma^p. \quad (\text{B.2})$$

$$\begin{aligned}
P_{out} &= \int_0^\infty \left[ 1 - e^{-(J_R - 1 + J_R \gamma)/\tilde{\alpha}_1} \sum_{l=0}^{N_1-1} \frac{1}{l!} \left( \frac{(J_R - 1 + J_R \gamma)}{\tilde{\alpha}_1} \right)^l \right]^Q f_{\gamma_2^{\min, \max}}(\gamma) d\gamma \\
&= \sum_{q=0}^Q \binom{Q}{q} (-1)^q \int_0^\infty e^{-q(J_R - 1 + J_R \gamma)/\tilde{\alpha}_1} \underbrace{\left[ \sum_{l=0}^{N_1-1} \frac{1}{l!} \left( \frac{(J_R - 1 + J_R \gamma)}{\tilde{\alpha}_1} \right)^l \right]^q}_{J_1} f_{\gamma_2^{\min, \max}}(\gamma) d\gamma.
\end{aligned} \tag{B.1}$$

Substituting (B.2) into (B.1), yields

$$\begin{aligned}
P_{out} &= \sum_{q=0}^Q \binom{Q}{q} (-1)^q e^{-\frac{q(J_R - 1)}{\tilde{\alpha}_1}} \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \\
&\quad \frac{\sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R - 1)^{\tilde{L}_1 - p} (J_R)^p}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\
&\quad \int_0^\infty e^{-q J_R \gamma / \tilde{\alpha}_1} \gamma^p f_{\gamma_2^{\min, \max}}(\gamma) d\gamma.
\end{aligned} \tag{B.3}$$

Again using (8) into (B.3), we have (B.4) at the top of the next page which proves (13).

#### APPENDIX C: A DETAILED DERIVATION OF THEOREM 2

Applying the Taylor series expansion truncated to the  $N_1$ th order given by  $e^x = \sum_{l=0}^{N_1} \frac{x^l}{l!} + O(x^{N_1})$ , we derive the first order expansion of  $F_{\gamma_1^{k^*, q^*}}(x)$ , which is specified in (5), at high  $\tilde{\alpha}_1$  as

$$\begin{aligned}
F_{\gamma_1^{k^*, q^*}}(x) &= \left[ 1 - e^{-x/\tilde{\alpha}_1} \left( e^{x/\tilde{\alpha}_1} - \frac{1}{N_1!} \left( \frac{x}{\tilde{\alpha}_1} \right)^{N_1} - O\left( \left( \frac{x}{\tilde{\alpha}_1} \right)^{N_1} \right) \right) \right]^Q \\
&= \frac{1}{(N_1!)^Q} \left( \frac{x}{\tilde{\alpha}_1} \right)^{QN_1} + O((\tilde{\alpha}_1)^{-QN_1}).
\end{aligned} \tag{C.1}$$

In addition, the PDF expression  $f_{\gamma_2^{\min, \max}}(x)$  in (8) needs to be written as

$$f_{\gamma_2^{\min, \max}}(x) = \hat{C} \sum \frac{x^{\tilde{N}_2 - 1}}{(\tilde{\alpha}_2)^{\tilde{N}_2}} e^{-\frac{\hat{\beta} x}{\tilde{\alpha}_2}} U(x). \tag{C.2}$$

Substituting (C.1) and (C.2) into (12), the asymptotic secrecy outage probability is calculated as (C.3) at the top of the next page which proves (15).

#### APPENDIX D: A DETAILED DERIVATION OF COROLLARY 1

The CDF of  $\gamma_2^{\min, \max}$  is given by

$$\begin{aligned}
F_{\gamma_2^{\min, \max}}(x) &= 1 - (1 - F_{\gamma_2^{k, \max}}(x))^K \\
&= 1 - \sum_{k=0}^K \sum_{m=0}^{Nk} \binom{K}{k} \binom{Nk}{m} (-1)^{k+m} e^{-mx/\tilde{\alpha}_2} \\
&\quad \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \frac{x^{\sum_{t=0}^{N_2-1} t v_{t+1}}}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}}.
\end{aligned} \tag{D.1}$$

In addition, the PDF of  $\gamma_1^{k^*, q^*}$  is given by

$$\begin{aligned}
f_{\gamma_1^{k^*, q^*}}(x) &= \frac{Q}{(\tilde{\alpha}_1)^{N_1} (N_1 - 1)!} \sum_{q=0}^{Q-1} \binom{Q-1}{q} (-1)^q \\
&\quad \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\
&\quad x^{N_1 + \sum_{t=0}^{N_1-1} t w_{t+1} - 1} e^{-\frac{x(q+1)}{\tilde{\alpha}_1}} U(x).
\end{aligned} \tag{D.2}$$

The probability of non-zero achievable secrecy rate is given by

$$\begin{aligned}
Pr(C_s > 0) &= \int_0^\infty F_{\gamma_2^{\min, \max}}(x) f_{\gamma_1^{k^*, q^*}}(x) dx \\
&= 1 - \frac{Q}{(\tilde{\alpha}_1)^{N_1} (N_1 - 1)!} \sum_{k=0}^K \sum_{m=0}^{Nk} \sum_{q=0}^{Q-1} \binom{Q-1}{q} \binom{K}{k} \binom{Nk}{m} \\
&\quad (-1)^{q+k+m} \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \\
&\quad \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\
&\quad \int_0^\infty e^{-x(\frac{m}{\tilde{\alpha}_2} + \frac{q+1}{\tilde{\alpha}_1})} x^{\tilde{N}_1 - 1} dx
\end{aligned} \tag{D.3}$$

which becomes (18).

#### APPENDIX E: A DETAILED DERIVATION OF COROLLARY 3

Based on (D.1), we first rewrite the CDF of  $\gamma_2^{\min, \max}$  as

$$F_{\gamma_2^{\min, \max}}(x) = 1 + \tilde{F}_{\gamma_2^{\min, \max}}(x), \tag{E.1}$$

where

$$\begin{aligned}
\tilde{F}_{\gamma_2^{\min, \max}}(x) &= \sum_{k=1}^K \sum_{m=1}^{Nk} \binom{K}{k} \binom{Nk}{m} (-1)^{k+m+1} e^{-mx/\tilde{\alpha}_2} \\
&\quad \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \frac{x^{\sum_{t=0}^{N_2-1} t v_{t+1}}}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}}.
\end{aligned}$$

Then, the ergodic secrecy rate is derived as (E.2) at the top of the next page. As  $\tilde{\alpha}_1 \rightarrow \infty$ ,  $\Theta_1$  asymptotically becomes

$$\begin{aligned}
\Theta_1^\infty &= \int_0^\infty \left[ \log(\tilde{\alpha}_1) + \log\left(\frac{x_1}{\tilde{\alpha}_1}\right) \right] f_{\gamma_1^{k^*, q^*}}(x_1) dx_1 \\
&= \log(\tilde{\alpha}_1) + \int_0^\infty \log\left(\frac{x_1}{\tilde{\alpha}_1}\right) f_{\gamma_1^{k^*, q^*}}(x_1) dx_1.
\end{aligned} \tag{E.3}$$

Substituting the PDF of  $\gamma_1^{k^*, q^*}$  given in (D.2) into (E.3), and employing [36, eq. 4.352.1] given by

$$\begin{aligned}
P_{out} &= C \widetilde{\sum}_{q=0}^Q \binom{Q}{q} (-1)^q e^{-\frac{q(J_R-1)}{\tilde{\alpha}_1}} \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{\sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R-1)^{\tilde{L}_1-p} (J_R)^p}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\
&\quad \int_0^\infty e^{-\gamma \left( \frac{q J_R}{\tilde{\alpha}_1} + \beta_2 \right)} \gamma^{p+\tilde{N}_2-1} d\gamma \\
&= C \widetilde{\sum}_{q=0}^Q \binom{Q}{q} (-1)^q e^{-\frac{q(J_R-1)}{\tilde{\alpha}_1}} \sum_{w_1, \dots, w_{N_1}}^q \frac{q!}{w_1! \dots w_{N_1}!} \frac{1}{\prod_{t=0}^{N_1-1} (t! (\tilde{\alpha}_1)^t)^{w_{t+1}}} \\
&\quad \sum_{p=0}^{\tilde{L}_1} \binom{\tilde{L}_1}{p} (J_R-1)^{\tilde{L}_1-p} (J_R)^p \left( \frac{q J_R}{\tilde{\alpha}_1} + \beta_2 \right)^{-(p+\tilde{N}_2)} (p+\tilde{N}_2-1)!.
\end{aligned} \tag{B.4}$$


---

$$\begin{aligned}
P_{out}^\infty &= \frac{\hat{C}}{(N_1!)^Q} \widehat{\sum} \int_0^\infty \left( \frac{J_R \gamma + J_R - 1}{\tilde{\alpha}_1} \right)^{Q N_1} \frac{\gamma^{\tilde{N}_2-1}}{(\tilde{\alpha}_2)^{\tilde{N}_2}} e^{-\frac{\hat{\beta} \gamma}{\tilde{\alpha}_2}} d\gamma + O((\tilde{\alpha}_1)^{-Q N_1}) \\
&= \frac{\hat{C}}{(N_1!)^Q} \widehat{\sum} \sum_{l=0}^{Q N_1} \binom{N_1}{l} \left( \frac{1}{\tilde{\alpha}_1} \right)^{Q N_1} (J_R-1)^{Q N_1-l} (J_R)^l \int_0^\infty \gamma^l \frac{\gamma^{\tilde{N}_2-1}}{(\tilde{\alpha}_1)^{\tilde{N}_2}} e^{-\frac{\hat{\beta} \gamma}{\tilde{\alpha}_2}} d\gamma + \\
&\quad O((\tilde{\alpha}_1)^{-Q N_1}) \\
&= \frac{C}{(N_1!)^Q} \widehat{\sum} \sum_{l=0}^{Q N_1} \binom{Q N_1}{l} \left( \frac{1}{\tilde{\alpha}_1} \right)^{Q N_1} (J_R-1)^{Q N_1-l} (J_R)^l (\tilde{\alpha}_2)^l \frac{(l+\tilde{N}_2-1)!}{(\hat{\beta})^{l+\tilde{N}_2}} + O((\tilde{\alpha}_1)^{-Q N_1}) \\
&= (G_a \tilde{\alpha}_1)^{-Q N_1} + O((\tilde{\alpha}_1)^{-Q N_1}).
\end{aligned} \tag{C.3}$$


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$$\begin{aligned}
\bar{C}_s &= \frac{1}{2 \log(2)} \int_0^\infty \left[ \int_0^{x_1} \frac{F_{\gamma_2^{\min, \max}}(x_2)}{1+x_2} dx_2 \right] f_{\gamma_1^{k^*, q^*}}(x_1) dx_1 \\
&= \frac{1}{2 \log(2)} \left[ \underbrace{\int_0^\infty \log(1+x_1) f_{\gamma_1^{k^*, q^*}}(x_1) dx_1}_{\Theta_1} + \underbrace{\int_0^\infty \int_0^{x_1} \frac{\tilde{F}_{\gamma_2^{\min, \max}}(x_2)}{1+x_2} f_{\gamma_1^{k^*, q^*}}(x_1) dx_2 dx_1}_{\Theta_2} \right].
\end{aligned} \tag{E.2}$$


---

$\int_0^\infty x^{\nu-1} e^{-\mu x} \log(x) dx = \frac{1}{\mu^\nu} \Gamma(\nu) [\psi(\nu) - \log(\mu)]$ , asymptotic expression for  $\Theta_2$  is given by we compute (E.3) as

$$\begin{aligned}
\Theta_1^\infty &= \log(\tilde{\alpha}_1) + \frac{Q}{(N_1-1)!} \sum_{q=0}^{Q-1} \binom{Q-1}{q} (-1)^q \\
&\quad \sum_{w_1, \dots, w_{N_1}}^q \left( \frac{q!}{w_1! \dots w_{N_1}!} \right) \frac{1}{\prod_{t=0}^{N_1-1} (t!)^{w_{t+1}}} \\
&\quad \frac{\Gamma(N_1 + \tilde{L}_1)}{(q+1)^{N_1 + \tilde{L}_1}} [\psi(N_1 + \tilde{L}_1) - \log(q+1)].
\end{aligned} \tag{E.4}$$

Changing the order of integration in  $\Theta_2$ , we have

$$\Theta_2 = \int_0^\infty \frac{\tilde{F}_{\gamma_2^{\min, \max}}(x_2)}{1+x_2} (1 - F_{\gamma_1^{k^*, q^*}}(x_2)) dx_2. \tag{E.5}$$

According to the first order expansion of the CDF of  $\gamma_1^{k^*, q^*}$  shown in (C.1), as  $\tilde{\alpha}_1 \rightarrow \infty$ ,  $F_{\gamma_1^{k^*, q^*}}(x_2) \approx 0$ . Hence, the

$$\begin{aligned}
\Theta_2^\infty &= \int_0^\infty \frac{\tilde{F}_{\gamma_2^{\min, \max}}(x_2)}{1+x_2} dx_2 \\
&= \sum_{k=1}^K \sum_{m=1}^{Nk} \binom{K}{k} \binom{Nk}{m} (-1)^{k+m+1} \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \\
&\quad \frac{1}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \int_0^\infty \frac{e^{-m x_2 / \tilde{\alpha}_2} x_2^{\sum_{t=0}^{N_2-1} v_{t+1}}}{1+x_2} dx_2 \\
&= \sum_{k=1}^K \sum_{m=1}^{Nk} \binom{K}{k} \binom{Nk}{m} (-1)^{k+m+1} \sum_{v_1, \dots, v_{N_2}}^m \left( \frac{m!}{v_1! \dots v_{N_2}!} \right) \\
&\quad \frac{\Gamma\left(\sum_{t=0}^{N_2-1} t v_{t+1} + 1\right)}{\prod_{t=0}^{N_2-1} (t! (\tilde{\alpha}_2)^t)^{v_{t+1}}} \\
&\quad \Psi\left(\sum_{t=0}^{N_2-1} t v_{t+1} + 1, \sum_{t=0}^{N_2-1} t v_{t+1} + 1; m/\tilde{\alpha}_2\right).
\end{aligned} \tag{E.6}$$

Substituting (E.6) and (E.4) into (E.2), we derive the asymptotic expression for the ergodic secrecy capacity as (26).

#### APPENDIX F: A DETAILED DERIVATION OF COROLLARY 4

In the case of  $\tilde{\alpha}_1 \rightarrow \infty$  and  $\tilde{\alpha}_2 \rightarrow \infty$  with  $\frac{\tilde{\alpha}_1}{\tilde{\alpha}_2} = \kappa$ , the asymptotic ergodic secrecy rate can be easily obtained based on the proof of Corollary 3 in Appendix E. We only need to further provide an asymptotic expression for  $\Theta_2^\infty$  with  $\tilde{\alpha}_2 \rightarrow \infty$ . Observing  $\Theta_1^\infty$  in (E.3), an asymptotic expression for  $\Theta_2^\infty$  can be derived as

$$\Theta_{21}^\infty = -\log(\tilde{\alpha}_2) - \int_0^\infty \log\left(\frac{x_2}{\tilde{\alpha}_2}\right) f_{\gamma_2^{\min, \max}}(x_2) dx_2. \quad (\text{F.1})$$

Substituting the PDF of  $\gamma_2^{\min, \max}$  in (8) into (F.1), we obtain

$$\begin{aligned} \Theta_{21}^\infty &= -\log(\tilde{\alpha}_2) - \hat{C} \sum \int_0^\infty e^{-\hat{\beta}x} x^{\tilde{N}_2-1} \log(x_2) dx_2 \\ &= -\log(\tilde{\alpha}_2) - \hat{C} \sum \frac{\Gamma(\tilde{N}_2)}{(\hat{\beta})^{\tilde{N}_2}} [\psi(\tilde{N}_2) - \log(\hat{\beta})]. \end{aligned} \quad (\text{F.2})$$

Substituting the new asymptotic expression for  $\Theta_2^\infty$  in (F.2) and (E.4) into (E.2), we get (29).

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